



Skimming Properties of Sharp-Crested Weirs

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Abstract: Weirs are structures that allow uppermost layers of liquids to flow by gravity downstream. They are, therefore, inherently gravity skimmers, and have successfully been used as skimmers in numerous applications, including oil-spill cleanup and various stormwater and wastewater treatment operations. This study examines the hydraulic skimming properties of sharp-crested weirs. The skimming properties of basic rectangular, triangular, and trapezoidal weirs are first characterized with a nondimensional skimming function. The triangular V-notch weir is shown to be a superior skimmer when compared to the other basic weirs. The V-notch weir has a maximum skimming rate $q'_m = 1.44$, occurring at an elevation $y'_m = \frac{2}{3}$. Considerable enhancements of skimming properties are, however, achieved with higher order polynomial weirs. Polynomial weirs of order n > 1 have maximum skimming rates $q'_m > 1.44$, occurring at desirable elevations $y'_m > \frac{2}{3}$. The results of this work can be applied to design high performing sharp-crested skimmer weir outlets. **DOI: 10.1061/(ASCE)IR.1943-4774.0001500.** © 2020 American Society of Civil Engineers.

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Introduction

Weirs can be defined as *gravity skimmers*, capable of discharging, through their openings, surface water, floating liquids, and debris. Fixed, movable, and floating skimmer weirs have been widely used in oil-spill cleanup operations (NOAA 2010) and the removal of scums and floating materials in water and wastewater treatment operations.

The impact of urbanization on increased runoff and pollution in streams is a well-recognized drainage problem (USEPA 1997). Among the critical concerns are issues related to water quantity (floods) and water quality (sedimentation and pollution). Best managements practices (BMPs) have been developed and investigated by planners and engineers to address these drainage issues. Huber (2006) has provided a comprehensive review of methodologies to evaluate the effectiveness of BMPs. There is a need to analyze and monitor both short and long-term effectiveness of BMPs (Liu et al. 2017). Of interest to this work is the performance of hydraulic devices and skimmers installed at the outlet of sedimentation reservoirs and stormwater ponds (Toso and Peterson 2014; Washington State DOT 2019).

Fixed and floating skimming devices are presently playing an important role in retaining sediments and pollutants by selectively withdrawing clearer water from the upper layers of impoundments (Jarrett 2001; Millen et al. 1997). Sedimentation routing models by Ward et al. (1979), Wilson and Barfield (1985), and others have also confirmed the benefits of skimmers in increasing the trapping efficiency of sedimentation facilities. Field observations showed that floating skimmers help retain higher percentages of sediments, provided they are prevented, during dewatering cycles, from sinking close to layers saturated with sediments (McCaleb and McLaughlin 2008). Periodic cleanup of floating skimmers

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and frequent sediment removal from the bottom of sediment basins are, therefore, essential practices (Zech et al. 2014).

Cox et al. (2014, 2015) and MacKenzie (2016) designed an elliptical weir to address clogging and maintenance issues of perforated risers and orifice plate outlets. Pilon et al. (2016) has also proposed a fixed skimmer made of two concentric perforated risers.

Despite the wide range of skimmer applications, the properties of skimmer weirs of different geometries and sizes have not been studied. This work was carried out to fill this gap and study the skimming properties of sharp-crested weirs.

The skimming properties of rectangular, triangular, and trapezoidal weirs are first analyzed in this paper using hydraulic principles. High order polynomial weirs are then examined and shown to have enhanced skimming properties.

Recent applications of polynomial weirs include the redesign of compound weirs to eliminate transitional hydraulic instabilities (Baddour 2019). A multistage polynomial weir system was also designed to replace stop-log operations at the outlet of Halfway Lake, Ontario, Canada (Gholmareza-Kashi and Baddour 2018). Polynomial weirs can now be precisely manufactured due to recent advancements in material cutting technologies, including laser, plasma, and water jet (Kalpakjian and Schmid 2017).

Hydraulics of Skimmer Weirs

Fig. 1 defines a sharp-crested weir, which allows a discharge Q to freely flow by gravity downstream, when the head of water upstream measured above the base of the crest is h. Note in Fig. 1, the maximum head of water = H, the top width of the weir $= (a_0 + B)$, and the bottom width of the weir $= a_0$. The skimming properties of the weir are analyzed in this study in terms of the skimming function

$$q(y) = Cb(y)U(y) \tag{1}$$

where U(y) = discharge velocity as a function of the vertical coordinate y, and b(y) = width of the weir opening, which, in general, is also a function of y. The discharge coefficient C is assumed to be constant. Physically, q is the discharge through the weir per unit height. Applying Bernoulli's equation, the discharge velocity is

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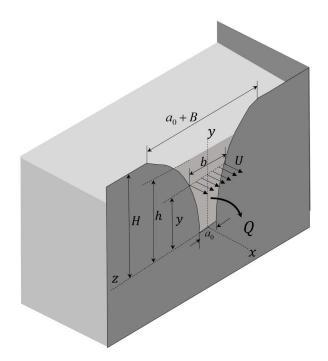


Fig. 1. Definition sketch of skimmer weir.

$$U = \sqrt{2g(h - y)} \tag{2}$$

where g is gravity, and the skimming function becomes

$$q = Cb\sqrt{2g(h-y)} \tag{3}$$

There are well-known limitations when applying Bernoulli's equation. More importantly, the flow is considered steady and both viscous and surface tension effects are assumed negligible. These limitations are consistent with common hydraulic derivations for sharp-edged weir discharge equations.

The discharge Q, through the opening of the weir, is the summation of elementary flows dQ = qdy in the range from y = 0 to y = h, or by integration

$$Q = \int_{y=0}^{h} q dy \tag{4}$$

It is convenient to generalize the weir skimming properties by normalizing the skimming function using two global weir parameters (namely, Q and h). Accordingly, on dimensional grounds, a normalized skimming function is defined as

$$q' = \frac{qh}{Q} \tag{5}$$

and a normalized vertical coordinate is

$$y' = \frac{y}{h} \tag{6}$$

It is interesting to note the area of the skimming function q' in the range y'=0 to 1 is exactly equal to unity. And this property applies to any shape and size of weir, since

$$\int_{y'=0}^{1} q' dy' = \int_{y=0}^{h} \left(\frac{qh}{Q}\right) \left(\frac{dy}{h}\right) = \frac{\int_{y=0}^{h} q dy}{Q} = 1$$
 (7)

In addition, the skimming function q', as defined in Eq. (5), is independent of the discharge coefficient C, which cancels out in this normalizing scheme.

Skimming Properties of Basic Weirs

Consider first the skimming behavior of basic rectangular and triangular weirs. For a rectangular weir the width $b = a_0$ = constant, and the head-discharge equation is $Q = \frac{2}{3}C\sqrt{2g}a_0h^{3/2}$. Hence, the skimming function defined in Eq. (5) for a rectangular weir is

$$q' = \frac{\sqrt{1 - y'}}{\beta_0}$$
 and $\beta_0 = \frac{2}{3}$ (8)

This function, plotted in Fig. 2, has a largest skimming rate q' = 1.5 at y' = 0. In comparison, the skimming function for a triangular V-notch weir, for which $b = a_1 y$ and $Q = \frac{4}{15} C a_1 \sqrt{2g} h^{5/2}$, is

$$q' = \frac{y'\sqrt{1-y'}}{\beta_1}$$
 and $\beta_1 = \frac{4}{15}$ (9)

According to Fig. 1, $a_1 = B/H$. This skimming function for the V-notch weir has a well-defined maximum skimming rate $q'_m = \frac{5}{2\sqrt{3}} = 1.44$ at a more desirable higher elevation $y'_m = \frac{2}{3}$. On the other hand, for an inverted V-notch weir, for which $b = a_0 - \frac{a_0}{H}y$ and $Q = C\sqrt{2g}(\beta_0 a_0 h^{3/2} - \beta_1 \frac{a_0}{H} h^{5/2})$, the skimming function is

$$q' = \frac{\left(1 - y'\frac{h}{H}\right)\sqrt{1 - y'}}{\beta_0 - \beta_1\frac{h}{H}} \tag{10}$$

which is showing a dependence on the relative head (h/H). As $(h/H) \to 0$, the behavior is clearly identical to the rectangular weir [i.e., Eq. (8)], and at full head as $(h/H) \to 1$, the largest skimming rate q' = 2.5 is occurring at y' = 0.

Eqs. (8) and (9) shown in Fig. 2 are universal and valid for any size of rectangular and V-notch weirs, and for any value of head h between 0 and H. It is not the case for the inverted V-notch [Eq. (10)] and trapezoidal weirs.

For trapezoidal weirs defined by $b=a_0+a_1y$ and $Q=C\sqrt{2g}(\beta_oa_oh^{3/2}+\beta_1a_1h^{5/2})$, the skimming function is

$$q' = \frac{\left(\frac{a_0}{B} + y'\left(\frac{h}{H}\right)\right)\sqrt{1 - y'}}{\beta_0\left(\frac{a_0}{B}\right) + \beta_1\left(\frac{h}{H}\right)}$$
(11)

where a_0 is bottom width, (a_0+B) is top width, and H is maximum head. Eq. (11) in Fig. 3 shows the development of the skimming function of a trapezoidal weir for a sequence of relative heads (h/H=0.1,0.4, and 1). It can be seen in Fig. 3 that the skimming function of a trapezoidal weir lies between the rectangular and V-notch weirs and the maximum skimming always occurs at a height $y_m' < \frac{2}{3}$. It is also interesting to note that the skimming functions of all the basic weirs in Figs. 2 and 3 are passing through the same point A with fixed coordinates

$$A\left(q_A' = \frac{\sqrt{\beta_0 - \beta_1}}{\beta_0^{3/2}} = 1.16, y_A' = \frac{\beta_1}{\beta_0} = 0.40\right)$$
 (12)

The maximum skimming height $y'_m = \frac{y_m}{h}$, at which the skimming rate is maximized, is a relevant parameter when designing a skimmer weir. By differentiating Eq. (11) and equating $\frac{dq'}{dy'}$ to zero, we obtain for a trapezoidal weir

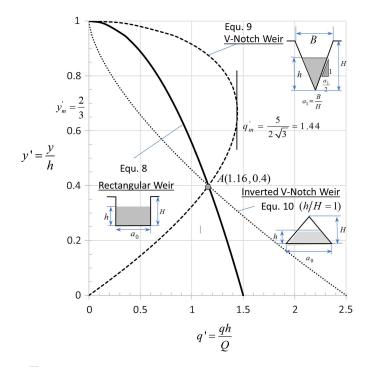


Fig. 2. Skimming functions for rectangular and triangular weirs.

$$y'_{m} = \frac{2}{3} - \left(\frac{a_{0}/B}{h/H}\right) \tag{13}$$

Eq. (13) is plotted in Fig. 4. As expected, when the size of the bottom width of the trapezoidal weir, $\frac{a_0}{B} \to 0$, the shape of the weir becomes a V-notch and the height of maximum skimming $y_m' \to \frac{2}{3}$. It is desirable for skimmer weirs to have values of y_m' as close as possible to 1 (i.e., maximum skimming occurring as close as possible to the free surface). Higher order weirs are proposed in the following to achieve this skimming objective with values of $y_m' > \frac{2}{3}$ and approaching 1.

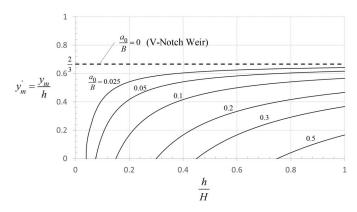


Fig. 4. Height of maximum skimming for trapezoidal weirs.

Skimming Properties of High Order Polynomial Weirs

The design of a high order polynomial skimmer weir is illustrated in Fig. 5, where the opening of the weir is given by

$$b = a_0 + a_n y^n \tag{14}$$

Here, a_0 is the weir opening at y=0, n is the order of the polynomial, and a_n is the polynomial constant. As defined in Fig. 5, since (a_0+B) is the top width of the weir at y=H, the polynomial constant must be

$$a_n = \frac{B}{H^n} \tag{15}$$

According to the general polynomial weir formulation (Baddour 2008), the discharge equation for the weir given in Eq. (14) is

$$Q = C\sqrt{2g}(\beta_0 a_0 h^{\frac{3}{2}} + \beta_n a_n h^{n+\frac{3}{2}})$$
 (16)

where C is the weir discharge coefficient and h is the head of water upstream measured above the crest. The constant $\beta_0 = \frac{2}{3}$ and the

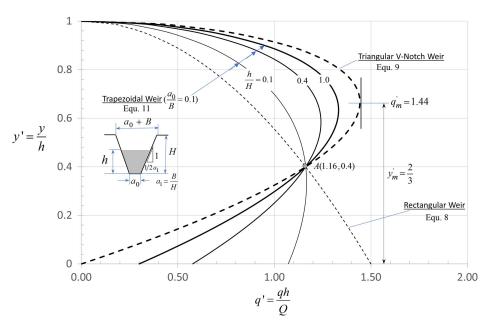


Fig. 3. Skimming functions for trapezoidal weir at different head of water.

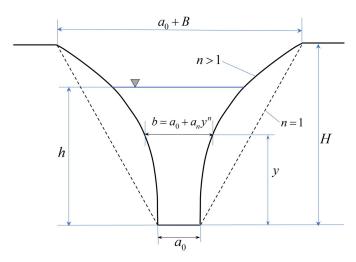


Fig. 5. Design of polynomial skimmer weir.

constant β_n can be determined with the following recurrent formula (Baddour 2019):

$$\beta_n = \beta_{n-1} \frac{2n}{2n+3} \tag{17}$$

The values of β_n are summarized in Table 1.

Consider now the skimmer weir given in Eq. (14) without an opening at y = 0 (i.e., $a_0 = 0$). In this case, the width of the weir is $b = a_n y^n$ and the weir skimming function defined in Eq. (5) reduces to

$$q' = \frac{y'^n \sqrt{1 - y'}}{\beta_n} \tag{18}$$

This skimming function is universal and applicable to this type of weir of any size. It does not depend on C and neither on h/H nor B/H. It is plotted in Fig. 6 for values of n=1 to 5. Clearly, it can be seen in Fig. 6 that the ability of this type of weir to skim water from layers closer to the free surface is enhanced as n increases from 1 to 5. More specifically, by differentiating Eq. (18) and setting dq'/dy'=0, the maximum skimming conditions are

$$y_m' = \frac{2n}{2n+1} \tag{19}$$

and

$$q'_{m} = \frac{(2n)^{n}}{\beta_{n}(2n+1)^{n+\frac{1}{2}}}$$
 (20)

For n = 1, the weir reduces to a V-notch and, as expected, $y'_m = \frac{2}{3}$ and $q'_m = 1.44$, as derived previously in the paper. The skimming

Table 1. Values of constant β_n in the discharge equation of polynomial weirs

n	eta_n
0	2/3
1	4/15
2	16/105
3	32/315
4	256/3,465
5	2,560/45,045

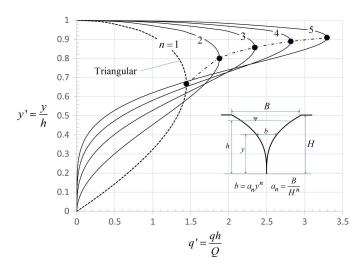


Fig. 6. Skimming function of polynomial weir without opening at y = 0.

properties of polynomial weirs for n=1 to 5 are summarized in Table 2.

As an example, when n=4, the maximum skimming rate $q_m'=2.817$ is notably 95% higher than the V-notch weir maximum skimming rate of 1.443. Also, this maximum skimming rate is occurring at a distance of (1-8/9)h, or 0.11h from the free surface, compared to a distance of 0.33h for a V-notch. This polynomial weir has, therefore, much superior skimming properties when compared to the V-notch weir.

The skimming function introduced in this paper defines the amount of water withdrawn from different elevations above the crest of the weir. The results have shown that by changing the weir geometry to a polynomial shape, it is possible to withdraw more water from elevations closer to the surface, which is the main objective of a skimmer. The polynomial weir design is, therefore, optimizing the shape of the skimming function.

Consider next the more general polynomial weir with a finite opening a_0 at y=0. A study of an elliptical weir (Cox et al. 2015) found that a small opening at y=0 is helping the passage of floating debris and facilitating manual cleanups. For the general polynomial weir case [Eq. (14)] with a finite value of a_0 , the skimming function is

$$q' = \frac{\left(\frac{a_0}{B} + y'^n \left(\frac{h}{H}\right)^n\right) \sqrt{1 - y'}}{\beta_0 \frac{a_0}{B} + \beta_n \left(\frac{h}{H}\right)^n}$$
(21)

This function is not universal, but dependent on the relative size of opening (a_0/B) , as well as the relative head (h/H). In Fig. 7, the plots of Eq. (21) are showing the evolution of the skimming function as (h/H) increases from 0.1 to 1 for a fourth-order polynomial weir with an opening $a_0/B=0.03$. Note at small heads, the

Table 2. Summary of skimming properties of polynomial weirs $(a_0 = 0)$

n	$y_m' = \frac{y}{h}$	$q_m'=rac{qh}{Q}$
1	2/3	1.443
2	4/5	1.878
3	6/7	2.343
4	8/9	2.817
5	10/11	3.294

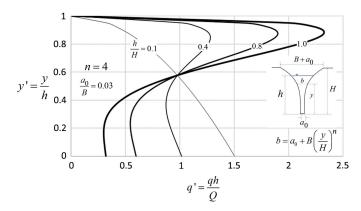


Fig. 7. Evolution of skimming function for polynomial weir with opening at y = 0.

skimming functions are like the skimming function of a rectangular weir [i.e., Eq. (8)]. On the other hand, at higher heads, maximum skimming rates from upper layers are gradually emerging. Understandably, when skimming from upper layers close to the surface is a design priority, the size of the opening a_0 is recommended to be kept to a minimum.

Comparison of Elliptical and Polynomial Weirs

Cox et al. (2014, 2015) studied the properties of sharp-edged elliptical weirs designed as water quality outlets for stormwater ponds. The elliptical weir was designed as an alternative to perforated risers and orifice plates, which are prone to clogging and require frequent maintenance (MacKenzie 2016). In addition to facilitating the cleanup and maintenance operations, the elliptical weir was efficient in withdrawing water from the surface layers, which carry less sediments and pollutants.

A direct comparison of elliptical and polynomial weir geometries is given in Fig. 8. On the left of Fig. 8 is the elliptical weir defined as

$$\frac{b}{B} = 1 - \sqrt{1 - \left(\frac{y}{H}\right)^2} \tag{22}$$

and on the right are polynomial weirs, of order n = 1 to 5, defined as

$$\frac{b}{B} = \left(\frac{y}{H}\right)^n \tag{23}$$

An opening a_0 at y=0 is intentionally not included in this comparison because it has similar effects on both weirs. It can be seen in Fig. 8 that the geometry of the elliptical weir is comparable to a polynomial weir of order $n\approx 3$. Since there is no explicit exact head-discharge solution for the elliptical weir (Cox et al. 2014), a different dimensionless skimming function, which does not depend on the total discharge Q, is needed for this comparison. The normalized skimming function adopted is

$$q^* = \frac{q}{\sqrt{2g}h^{1/2}} \tag{24}$$

which is independent of Q, but as will be seen next, varies with the discharge coefficient C. This dependence on C is not a serious problem because, here, we are only comparing weirs of similar geometries, which are expected to have comparable values of C.

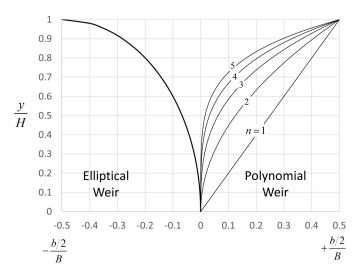


Fig. 8. Geometry of elliptical and polynomial weirs.

The skimming functions according to Eq. (24) for elliptical and polynomial weirs are, respectively

Elliptical Weir:
$$q^* = C_E \sqrt{1-y'} \left(1-\sqrt{1-y'^2\left(\frac{h}{H}\right)^2}\right)$$
 (25)

Polynomial Weir:
$$q^* = C_P \sqrt{1 - y'} y'^n \left(\frac{h}{H}\right)^n$$
 (26)

where C_E and C_P are the discharge coefficients for elliptical and polynomial weirs. The discharge coefficient $C_E = 0.64$ (Cox et al. 2014), and we can assume that $C_E = C_P$. Note the skimming functions in Eqs. (25) and (26) are not universal but vary with the relative head (h/H).

Fig. 9 compares the skimming functions for elliptical and third-order polynomial weirs at mid head h/H=0.5 and full head h/H=1. As expected, both weirs have similar skimming properties. At full head, the polynomial weir is withdrawing more flow from layers close to the surface and less flow from layers close to the base.

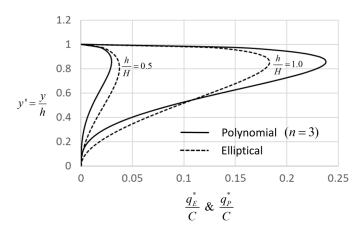


Fig. 9. Comparison of skimming properties of elliptical and third-order polynomial weirs.

Furthermore, since exact solutions of the head-discharge equations of polynomial weirs of any order can be readily obtained from Baddour (2008), it is not difficult to design polynomial weir outlets that provide any required dewatering times.

Concluding Remarks

Hydraulic outlets are playing an important role in determining the performance of BMPs, and surface skimmers are presently used in practice to control stormwater pollution and sedimentation. There is a need, however, to improve existing BMPs that require frequent labor-intensive maintenance. This goal can be achieved through rehabilitation and implementation of new outlet technologies. The polynomial skimmer weir described in this paper is one of these new technologies.

The skimming properties of sharp-crested weirs were analyzed in this study by applying basic hydraulic principles. These properties are presented in terms of nondimensional skimming functions that can be used to design skimmer weirs. In summary, the main conclusions are:

- 1. The V-notch weir is a superior skimmer compared to rectangular, inverted V-notch, and trapezoidal weirs, with $y'_m = \frac{2}{3}$ and $q'_m = 1.44$.
- 2. The skimming properties can be significantly enhanced with high order polynomial weirs. The higher the order, the closer to the surface is the skimmed layer. The skimming properties of polynomial weirs are summarized in Table 2.
- 3. The skimming properties of an elliptical weir are comparable to the properties of a third-order polynomial weir.
- Exact solutions of head-discharge equations for polynomial skimmer weirs of any order can be obtained using Eqs. (16) and (17).

The theory presented in this paper has considered only water passing through the weir. Future research is recommended to examine, experimentally, the effect of floating solids and debris and possible applications of polynomial weirs for either enhancing or blocking the passage of floating materials downstream.

Data Availability Statement

No data, models, or code were generated or used during the study.

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